Problem Set 5

Problem 1. Computations 1
Consider the one-sector growth model and suppose that $u(c,\ell) = \log c$ and that $F(k,n) = k^\alpha n^{1-\alpha}$. Write a code in MATLAB to solve the one-sector growth model using value function iterations. Assume that $\alpha = 1/3$, $\delta = 0.08$, and $\beta = 0.96$. Using your solution calculate the steady state level of capital. Using your computations and by referring to the notes, calculate the rate of convergence of this economy as in Barro and Sala-i-Martin's exercise.

Problem 2. Computations 2
Consider the stochastic one-sector growth model and suppose that $u(c,\ell) = \log c + \psi \log \ell$ and that $y_t = A_t k_t^\alpha n_t^{1-\alpha}$. Assume that $\alpha = 0.34$, $\beta = 0.96$ and $\delta = 0.08$.

a. If we were to calibrate ψ so that households on average work, 1/3 of their time, what value do you get for ψ by matching this statistic in the non-stochastic steady state of the model.

b. In this part, you will calibrate the process for TFP. To do so, you need to use a few macro data sources. Use data provided by Penn World Table for years 1950-2014 for Real GDP, Real Capital and Total Hours Worked for the United States. Calculate A_t as a residual using the formulation of the production function. In order to remove long-term changes from the series, de-trend the dataset using your filter of choice (linear trend, HP filter, Band-Pass filter). Using the de-trended series, run the following regression to estimate the process for A_t:

$$\log A_{t+1} = \rho \log A_t + \varepsilon_{t+1}$$

where $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$. Once you have estimated this, use the Tauchen-Hussey method to approximate this process using a discrete Markov process which takes three values. Your asnwer should specify grid points for $\log A_t$ as well as transition matrix. Given this process, solve the model using value function iteration in MATLAB or other programs.

c. Calculate the stationary distribution associated with the policy functions that you found in part b.

d. Using the stationary distribution that you found in part c, calculate standard deviation of GDP, consumption, investment, and hours. Discuss the results.

Problem 3. The Merton Problem
Consider a consumer who has standard CRRA utility function

$$\sum_{t=0}^{T} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$$

and lives for T periods. Suppose that in each period the consumer can invest in a risk-free asset with gross return R_f and a risky assets whose gross return is given by R_t which is an i.i.d. random variable distributed according to some continuous distribution $F(R)$. Suppose that initially, the consumer's wealth is given by W_0.

a. Write down the sequence of budget constraints for the consumer.

b. Formulate the consumer's problem recursively.

c. Solve the consumer's problem using a guess and verify method.

d. From part c, what does the portfolio choice of the consumer look like, i.e., what fraction of his/her investments in each period are in bonds vs stocks? How does this depend on age/time? Is this in line with the advice given by financial planners? If not, what ingredients can be added to the model to address this discrepancy?

Problem 4.
Solve problems 14.2, 14.3, 14.4, 14.8, 14.9, 14.10.

Problem 5. Asset Pricing with Recursive Preferences
Consider the standard stochastic growth model – assume exogenous labor supply – and suppose that instead of time-separable preferences, we use Epstein-Zin preferences given by

$$V_t\left(s^t\right) = C_t\left(s^t\right)^{1-\beta}\left[\sum_{s^{t+1}\succeq s^t}\pi\left(s^{t+1}|s^t\right)V_{t+1}\left(s^{t+1}\right)^{1-\gamma}\right]^{\frac{\beta}{1-\gamma}}$$

Assume that s_t is a First Order Markov process.

a. Calculate the intertemporal elasticity of substitution in this model. *Hint: You can use the deterministic version of the model to do so.*

b. Write down the planner's problem for this economy in sequence form and recursively.

c. Derive the Euler equation for capital in this model. Use the same logic that we discussed in our class to talk about what affects investment and how investment interacts with uncertainty.

d. Calculate the SDF associated with these preferences and use it to solve for price of a security with dividend process given by $\{d_t\left(s^t\right)\}_{\forall t, s^t\in\mathcal{S}^{t+1}}$.

e. Repeat a similar exercise that we did in class on equity premium for these preferences. That is assume that $\log c_t$ is a random walk – with normal innovations and a drift – while risky and safe returns are distributed normally and i.i.d. Solve for the equity premium. Can this model succesfully match the equity premium for reasonable parameter values?